Question 4

$H = X(X'X)^{-1}X'$

$\hat{Y} = HY$

Show that $H^2 = H$

Idempotent.

$H \cdot H = H$

$= X(X'X)^{-1}X'[X(X'X)^{-1}X']$

$= X(X'X)^{-1}(X'X)(X'X)^{-1}X'$

$= X(X'X)^{-1}X'$

$= H$

Show that:

$H(1-H) = 0$

$HH = H$

$= H - HH \Rightarrow$

$X(X'X)^{-1}X' - X(X'X)^{-1}X' = 0$

Show That.

$H' = H$

$H' = \left(X(XX')^{-1}X\right)'$

$= X\left[(X'X^{-1})^{-1}\right]'X'$

$= X(X'X)^{-1}X'$

$= \underline{\underline{H}}$

Show that

$(I - H)'H = 0$

$= (I' - H')H$

but $(I' - H') = (I - H)$

$(I - H)H = X(XX^{-1})^{-1}X' - X(XX^{-1})^{-1}X'$

$= \underline{\underline{0}}$

Question 4 (b)

Show that

$He = 0$

$e = Y - \hat{Y}$

$He = H(Y - \hat{Y})$

$\hat{e} = y - \hat{y} = y - Hy = (I - H)y$

but $(I - H) = 0$

$= 0y$

Therefore $= 0$

$He = 0$

It imply it is oblique projection

Question 4 (c)

(c)

$a'b = 0$ confirm that e & $\hat{Y}$ are perpendicolar.

perpendicolar if their dot product equals zero

Scanned with CamScanner

$e = Y - \hat{Y}$

$\hat{Y} = Y - e$

$= (Y - e)(Y - e)$

$= 0$

$= Y^2 - eY - eY + e^2$

$= Y^2 - 2eY + e^2$

$= 0 - 2(0) + 0$

$= 0$